LECTURE 9

(9-1)

Example 1

```
  Using Toolbox Path Cache.  Type "help toolbox_path_cache" for more info.

  To get started, select "MATLAB Help" from the Help menu.

>> A=[2 4 8 7 23;4 8 4 6 -13;5 7 3 2 1]

A =

     2     4     8     7    23
     4     8     4     6   -13
     5     7     3     2     1

>> A(1,:)=A(1,:)/A(1,1)

A =

    1.0000    2.0000    4.0000    3.5000   11.5000
    4.0000    8.0000    4.0000    6.0000  -13.0000
    5.0000    7.0000    3.0000    2.0000    1.0000

>> A(2,:)=A(2,:)-A(2,1)*A(1,:)

A =

    1.0000    2.0000    4.0000    3.5000   11.5000
         0         0  -12.0000   -8.0000  -59.0000
    5.0000    7.0000    3.0000    2.0000    1.0000

>> A(3,:)=A(3,:)-A(3,1)*A(1,:)

A =

    1.0000    2.0000    4.0000    3.5000   11.5000
         0         0  -12.0000   -8.0000  -59.0000
         0   -3.0000  -17.0000  -15.5000  -56.5000

>> temp=[0 0 0 0 0];
>> temp=A(2,:);
>> A(2,:)=A(3,:);
>> A(3,:)=temp;
>> A

A =

    1.0000    2.0000    4.0000    3.5000   11.5000
         0   -3.0000  -17.0000  -15.5000  -56.5000
         0         0  -12.0000   -8.0000  -59.0000

>> A(2,:)=A(2,:)/A(2,2)

A =

    1.0000    2.0000    4.0000    3.5000   11.5000
         0    1.0000    5.6667    5.1667   18.8333
         0         0  -12.0000   -8.0000  -59.0000

>> A(1,:)=A(1,:)-A(1,2)*A(2,:)
```

Interchanging the 2nd and 3rd row.

```
A =

    1.0000         0   -7.3333   -6.8333  -26.1667
         0    1.0000    5.6667    5.1667   18.8333
         0         0  -12.0000   -8.0000  -59.0000

>> A(3,:)=A(3,:)/A(3,3)

A =

    1.0000         0   -7.3333   -6.8333  -26.1667
         0    1.0000    5.6667    5.1667   18.8333
         0         0    1.0000    0.6667    4.9167

>> A(1,:)=A(1,:)-A(1,3)*A(3,:)

A =

    1.0000         0         0   -1.9444    9.8889
         0    1.0000    5.6667    5.1667   18.8333
         0         0    1.0000    0.6667    4.9167

>> A(2,:)=A(2,:)-A(2,3)*A(3,:)

A =

    1.0000         0         0   -1.9444    9.8889
         0    1.0000         0    1.3889   -9.0278
         0         0    1.0000    0.6667    4.9167

>>
```

Cartesian Eqn of Null space

Null space of A

$x = (x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5)$

$Ax = 0$

$$\Rightarrow \begin{aligned} x_1 - 1.94x_4 + 9.89x_5 &= 0 \\ x_2 + 1.39x_4 - 9.03x_5 &= 0 \\ x_3 + .67x_4 + 4.91x_5 &= 0 \end{aligned}$$

$$\begin{pmatrix} 1.94x_4 - 9.89x_5 \\ -1.39x_4 + 9.03x_5 \\ -.67x_4 - 4.91x_5 \\ x_4 \\ x_5 \end{pmatrix}$$

$$\begin{pmatrix} 1.94 \\ -1.39 \\ -.67 \\ 1 \\ 0 \end{pmatrix} x_4 + \begin{pmatrix} -9.89 \\ 9.03 \\ -4.91 \\ 0 \\ 1 \end{pmatrix} x_5$$

Basis of the Null space

Row space is spanned by the vectors.

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1.94 \\ 9.89 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1.39 \\ -9.03 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ .67 \\ 4.92 \end{pmatrix}$$

Cartesian Equation of the row space

$$\boxed{\begin{aligned} 1.94x_1 - 1.39x_2 - .67x_3 + x_4 &= 0 \\ -9.89x_1 + 9.03x_2 - 4.91x_3 + x_5 &= 0 \end{aligned}}$$

Dimension of the row space = 3.

Dimension of the null space = 2

$$\left.\begin{array}{l} \text{Null space is a 2-plane} \\ \text{Row space is a 3-plane} \end{array}\right\} \text{in } \mathbb{R}^5.$$

Example 2

```
>>
>> v1

v1 =

     1     5     6    -7     8    -9     5    12

>> v2

v2 =

     3     9   -12     8     6     1     5   -19

>> v3

v3 =

    -7   -11    78   -67    16   -49     5   136

>> v4

v4 =

    -5     5    -6     8     8    13   -11   -14

>> v5

v5 =

    93    39   -60    -8   -30  -145   187   -41
```

1
2
3
4
5

Check if these 5 vectors are independent.

Row Reduction

```
>> v2=v2-v2(1)*v1

v2 =

     0    -6   -30    29   -18    28   -10   -55

>> v3=v3-v3(1)*v1

v3 =

     0    24   120  -116    72  -112    40   220

>> v4=v4-v4(1)*v1

v4 =

     0    30    24   -27    48   -32    14    46
```

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```
>> v5=v5-v5(1)*v1

v5 =

         0      -426      -618       643      -774       692      -278
  -1157

>> v2=v2/v2(2)

v2 =

         0    1.0000    5.0000   -4.8333    3.0000   -4.6667    1.6667    9.1667

>> v3=v3-v3(2)*v2

v3 =

     0     0     0     0     0     0     0     0
```

The vector V_3 needs to be dropped.

```
>> v4=v4-v4(2)*v2

v4 =

     0     0  -126   118   -42   108   -36  -229

>> v5=v5-v5(2)*v2

v5 =

  1.0e+003 *

         0         0    1.5120   -1.4160    0.5040   -1.2960    0.4320    2.7480

>> v4=v4/v4(3)

v4 =

         0         0    1.0000   -0.9365    0.3333   -0.8571    0.2857    1.8175

>> v5=v5-v5(3)*v4

v5 =

  1.0e-012 *

         0         0         0         0         0   -0.2274         0   -0.4547
```

The vector V5 is numerically zero.

Approximating the vectors upto 3 places of decimal.

```
>> v1=round(1000*v1)/1000

v1 =

     1     5     6    -7     8    -9     5    12

>> v2=round(1000*v2)/1000

v2 =

         0    1.0000    5.0000   -4.8330    3.0000   -4.6670    1.6670    9.1670
```

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```
>> v3=round(1000*v3)/1000

v3 =

     0     0     0     0     0     0     0     0

>> v4=round(1000*v4)/1000

v4 =

         0         0    1.0000   -0.9370    0.3330   -0.8570    0.2860    1.8170

>> v5=round(1000*v5)/1000

v5 =

     0     0     0     0     0     0     0     0

>>
```

The vector V5 needs to be dropped.

Conclusion:

The vectors V1, V2 and V4 are linearly independent.

— x —.


```
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  To get started, select "MATLAB Help" from the Help menu.

>> v1=[1 5 6 -7 8 -9 5 12];
>> v2=[3 9 -12 8 6 1 5 -19];
>> v3=5*v1 - 4*v2;
>> v4=[-5 5 -6 8 8 13 -11 -14];
>> v5=11*v2 - 12*v4;
```

Here is the secret

Example 3

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`>> A=[1 5 ; 6 9]`

A =

$$\begin{pmatrix} 1 & 5 \\ 6 & 9 \end{pmatrix}$$

`>> B=[2 4 ; -45 29]`

B =

$$\begin{pmatrix} 2 & 4 \\ -45 & 29 \end{pmatrix}$$

`>> C=[40 -27 ; 36 -39]`

C =

$$\begin{pmatrix} 40 & -27 \\ 36 & -39 \end{pmatrix}$$

`>> D=[4 87;32 65]`

D =

$$\begin{pmatrix} 4 & 87 \\ 32 & 65 \end{pmatrix}$$

`>> H=[A B;C D]`

H =

$$\left(\begin{array}{cc|cc} 1 & 5 & 2 & 4 \\ 6 & 9 & -45 & 29 \\ \hline 40 & -27 & 4 & 87 \\ 36 & -39 & 32 & 65 \end{array}\right)$$

`>> h=det(H)`

h =

8338

`>> a=det(A)`

a =

-21

`>> P=D-C*(inv(A))*B`

P =

$$\begin{pmatrix} 540.1429 & -127.0476 \\ 554.4286 & -131.1429 \end{pmatrix} = D - CA^{-1}B$$

`>> p=det(P)`

p =

$$-397.0476 = \det(D - CA^{-1}B)$$

`>> hh=a*p`

hh =

8.3380e+003

$$[\det A][\det(D - CA^{-1}B)] = 8338$$

`>> h-hh`

ans =

-2.0009e-011

`>> round(1000*(h-hh))/1000`

ans =

0

We have verified the formula

$$\det H = [\det A][\det(D - CA^{-1}B)]$$

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```
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>> A=1

A =

     1

>> B=[5 2 4]

B =

     5     2     4

>> C=[6 ;40; 36]

C =

     6
    40
    36

>> D=[9 -45 29;-27 4 87;-39 32 65]

D =

     9   -45    29
   -27     4    87
   -39    32    65

>> P=A-B*(inv(D))*C

P =

    0.2737

>> d=det(D)

d =

       30462

>> h=d*P

h =

  8.3380e+003
```

$\boxed{\text{Example 4}}$

$$H = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Same as in example 3.

$$\det H = [\det D]\left[\det \underbrace{\left(A - B D^{-1} C\right)}_{= P \text{ a scalar}}\right]$$

$= 8338 \leftarrow$ we know that this is $\det H$.

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```
>> DD=d*inv(D)

DD =

  1.0e+003 *
```

This is [adj D]

$$\begin{pmatrix} -2.5240 & 3.8530 & -4.0310 \\ -1.6380 & 1.7160 & -1.5660 \\ -0.7080 & 1.4670 & -1.1790 \end{pmatrix}$$

```
>> A*det(D)-B*DD*C

ans =

        8338
```

= det H.

```
>>
```

When A is a scalar we have

$\det H = A\cdot[\det D] - B\,[\operatorname{adj} D]\,C$

X